THIN SHELLS ACTED ON BY BROADBAND RANDOM LOADS

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The study of the vibrations of thin shells acted on by broadband random loads presented considerable problems. In using the standard device of expanding the shell deflection in a series in the modes of the free vibrations [1], it is necessary to take into account a large number of vibrational modes. But with this approach such important quantities as the bending moment and deflection are represented as sums of large number of terms, and general conclusions regarding the effects of various problem parameters can only be obtained by tedious calculations. Some thoughts about the approximate computation of the defining sums for characteristic parameters are contained in [2 to 4]. Bolotin [2], for example, succeeded in obtaining relatively simple expressions for the spectral density of stresses in a plate for the case where the load is a three-dimensional white noise which is a stationary random function of time with arbirtary spectral density. An approximate calculation of a series for the case of a cylindrical shell is given in [4].

What follows is the solution of the problem of a normal load in the form of a spatial-temporal homogeneous random field. We limit our discussion to vibration of shells of sufficient extent with damping so that the disturbances introduced by the boundary conditions are significant at the edges of the shell only. This allows us to ignore boundary conditions completely in computing the probabilistic characteristics of deflection and acceleration at the points far removed from the edge. In determining the probabilistic characteristics near a particular shell edge, we take into account only the boundary conditions at the edge. Such a limitation simplifies substantially the computation of shell vibrations. These computations yield a number of new formulas, along with some known previously as special cases. Expansion of the shell deflection in a series in the vabrational modes is thereby circumvented.

1. By hypothesis, a large number of vibrational modes are excited in the shell, so that the stress condition in the latter has a large variability index. Let us make use of the appropriate equations of shell vibration [5 and 6],

$$D\left[1+R\left(\frac{d}{dt}\right)\right]\Delta\Delta w-\left(k_2\frac{\partial^2\varphi}{\partial x^2}+k_1\frac{\partial^2\varphi}{\partial y^2}\right)+\rho\frac{\partial^2 w}{\partial t^2}=p(t,\,x,\,y)$$

$$Eh\left[1+R\left(\frac{d}{dt}\right)\right]\left(k_2\frac{\partial^2 w}{\partial x^2}+k_1\frac{\partial^2 w}{\partial y^2}\right)+\Delta\Delta\varphi=0 \qquad (1.1)$$

where w is the normal shell deflection, p is the normal load, φ is the stress function for tangential forces, k_1 and k_2 are the principal curvatures of the shell, D is its cylindrical rigidity, h is the thickness, E is Young's modulus, and ρ is the linear mass of the shell. The resistance is assumed to be linear.

Equations (1.1) differ from the equations given in [6] in that the second term in the first and second equations allows for energy dissipation during vibration.

For simplicity, it is assumed in Equations (1.1) that $R(d/dt)$ is a polynomial containing only odd powers of the operator of differentiation with respect to time. It is assumed that energy dissipation is generally small, so that the maximum value of the second term is considerably larger than the maximum value of the first, for any motions of the shell.

Let us suppose that the normal load on the shell forms a homogeneous random field of zero mathematical expectation. We specify the load in terms of its spectral representation,

$$p=\iiint_{-\infty}^{\infty} e^{i(\omega t+\lambda x+\mu y)} V(\omega,\lambda,\mu)\,d\omega d\lambda d\mu \tag{1.2}$$

where V is a random function of the three-dimensional white noise type of intensity $S_p(\omega,\lambda,\mu)$. The nonrandom function S_p is called the spectral density of the load.

By virtue of the foregoing, all boundary conditions may be ignored in considering the behavior of the shell at points far removed from its edges. Then, introducing (1.2) into Equations (1.1), we find the spectral representation of the deflection at points sufficiently far from the edges,

$$w=\iiint_{-\infty}^{\infty} e^{i(\omega t+\lambda x+\mu y)}\frac{V(\omega,\lambda,\mu)\,d\omega d\lambda d\mu}{\Omega(\omega,\lambda,\mu)} \tag{1.3}$$

Here

$$\Omega(\omega,\lambda,\mu)=D_k\left[(\lambda^2+\mu^2)^2+\frac{Eh}{D}\frac{(k_2\lambda^2+k_1\mu^2)^2}{(\lambda^2+\mu^2)^2}\right]-\rho\omega^2 \tag{1.4}$$

$$D_k=D(1+i\psi),\qquad \psi(\omega)=-iR(i\omega) \tag{1.5}$$

In accordance with the above, the values of the function ψ turn out to be real and substantially less than unity. Using Expression (1.3) it is easy to find the correlation function of the deflection,

$$K_w(\tau,\xi,\eta)=\iiint_{-\infty}^{\infty} e^{i(\omega\tau+\lambda\xi+\mu\eta)}\frac{S_p(\omega,\lambda,\mu)\,d\omega d\lambda d\mu}{|\Omega(\omega,\lambda,\mu)|^2} \tag{1.6}$$

Formula (1.6) is too complex to be used for direct computations. It does, however, easily yield more compact probabilistic characteristics of deflection. Let us limit ourselves, for example, to computing the correlation fuction for $\xi=\eta=0$,

$$K_w(\tau)=K_w(\tau,0,0) \tag{1.7}$$

By the nature of the case, $K_w(\tau)$ is the deflection correlation function at one point of the shell. Combining (1.6) and (1.7), we easily find an expression for the spectral density of this displacement,

$$\Phi_w(\omega)=\iint_{-\infty}^{\infty}\frac{S_p(\omega,\lambda,\mu)\,d\lambda d\mu}{|\Omega(\omega,\lambda,\mu)|^2} \tag{1.8}$$

In some cases one must know the acceleration $w^{\cdot\cdot}$ of points on the shell. Its spectral density is obtained by multiplying (1.8) by ω^4.

$$\Phi_{w^{\cdot\cdot}}(\omega)=\iint_{-\infty}^{\infty}\frac{\omega^4 S_p(\omega,\lambda,\mu)\,d\lambda d\mu}{|\Omega(\omega,\lambda\mu)|^2} \tag{1.9}$$

2. Let us compute the spectral density of the acceleration at one point of the shell for various types of loads. A load of the wave type has a correlation function of the form

$$K_p(\tau,\xi,\eta)=K_p\left(\tau-\frac{\xi\cos\gamma}{c}-\frac{\eta\sin\gamma}{c}\right) \tag{2.1}$$

where c is the velocity of propagation of the pressure waves and γ is the angle between the wave vector and the x-axis. The spectral density of the load S_p defined by Formula

$$K_p(\tau, \xi, \eta) = \iiint\limits_{-\infty}^{\infty} e^{i(\omega\tau+\lambda\xi+\mu\eta)} S_p(\omega, \lambda, \mu)\, d\omega d\lambda d\mu \tag{2.2}$$

is of the form

$$S_p(\omega, \lambda, \mu) = \Phi_p(\omega)\, \delta\left(\lambda + \frac{\omega}{c}\cos\gamma\right) \delta\left(\mu + \frac{\omega}{c}\sin\gamma\right) \tag{2.3}$$

for the load (2.1).

Here $\Phi_p(\omega)$ is the spectral density of the pressure considered at one point of the shell. Introdicing (2.3) into (1.9) and computing the integral, we obtain the following relationship between the spectral densities of the acceleration and pressure at one point of the shell:

$$\Phi_{w^{\cdot\cdot}}(\omega) = \frac{\omega^4 \Phi_p(\omega)}{|D\ \ [(\omega/c)^4 + (Eh/D)(k_2\cos^2\gamma + k_1\sin^2\gamma)^2] - \rho\omega^2|^2} \tag{2.4}$$

Next let us suppose that the load is a three-dimensional white noise.. In this case its correlation function and spectral density are

$$K_p(\tau, \xi, \eta) = K_p(\tau)\,\delta(\xi)\,\delta(\eta), \qquad S_p(\omega, \lambda, \mu) = \Psi(\omega) \tag{2.5}$$

We introduce S_p as given by Formula (2.5) into (1.8),

$$\Phi_{w^{\cdot\cdot}}(\omega) = \Psi(\omega)\,\omega^4 \iint\limits_{-\infty}^{\infty} \frac{d\lambda d\mu}{|\Omega(\omega, \lambda, \mu)|^2} \tag{2.6}$$

The integral in this expression can be computed. The first step is to convert to the new integration variables z and θ related to the old ones by Expressions

$$\lambda = \sqrt{z}\cos\theta, \qquad \mu = \sqrt{z}\sin\theta \tag{2.7}$$

We now have

$$\Phi_{w^{\cdot\cdot}}(\omega) = \frac{\Psi(\omega)\,\omega^4}{2} \int\limits_0^{2\pi} d\theta \int\limits_0^{\infty} \frac{dz}{|D_k\,[z^2 + (Eh/D)(k_2\cos^2\theta + k_1\sin^2\theta)^2] - \rho\omega^2|^2} \tag{2.8}$$

To compute the integral over z in (2.8) we apply Formula [8]

$$\int\limits_0^{\infty} \frac{dz}{a + bz^2 + cz^4} = \frac{\pi}{2cq^2\sin\alpha}\ \frac{\cos{}^1/{}_2\,\alpha}{q} \tag{2.9}$$

$$q = (a/c)^{1/4}, \qquad \cos\alpha = -b/(2\sqrt{ac}) \qquad (0 \leqslant \alpha \leqslant \pi) \tag{2.10}$$

Combiming (2.9) and the integral over z in (2.8), we find that the constants a, b and c are as follows:

$$c = (1 + \psi^2) D^2, \qquad b = [2(d^2 - \delta^2) + 2d^2\psi^2]\, D, \qquad a = (d^2 - \delta^2)^2 + d^4\psi^2 \tag{2.11}$$

$$\delta^2 = \rho\omega^2, \qquad d^2 = d^2(\theta) = Eh\,(k_2\cos^2\theta + k_1\sin^2\theta)^2 \tag{2.12}$$

Computation of the first fraction in the right-hand side of Formula (2.9) yields

$$\frac{\pi}{2cq^2\sin\alpha} = \frac{\pi}{2\,|\psi|\,\rho\omega^2 D} \tag{2.13}$$

The second fraction yields an overly cumbersome expression, but is bounded for almost all values of ω as $\psi \to 0$. Hence, setting ψ small, we find its asymptotic representation as $\psi \to 0$. In this case Formulas (2.10) and (2.11) imply that

$$\cos\alpha = \sin(\delta - d), \qquad \cos\frac{\alpha}{2} = \begin{cases} 1 & \text{for } \delta > d, \\ 0 & \text{for } \delta < d, \end{cases} \qquad q = \frac{\sqrt{|d^2 - \delta^2|}}{\sqrt{D}} \tag{2.14}$$

Substituting (2.13) and (2.14) into (2.9), we obtain an approximate value

J_1 of the integral over z in Formula (2.8).

$$J_1 = \pi\,[2\,|\psi|\,\rho\,\sqrt{D\rho}\,\omega^2\,\sqrt{\omega^2 - \rho^{-1}d^2(\theta)}]^{-1} \quad \text{for } \omega^2 > \rho^{-1}d^2(\theta)$$
$$J_1 = 0 \qquad \text{for } \omega^2 < \rho^{-1}d^2(\theta) \tag{2.15}$$

Introducing this expression into Formula (2.8), we finally find that

$$\Phi_{w^{\cdot\cdot}}(\omega) = \frac{\pi^2\Psi(\omega)\,|\omega|}{2\,|\psi|\,\rho\,\sqrt{D\rho}}\,H(\alpha,\chi) \tag{2.16}$$

where

$$H(\alpha,\chi) = \frac{2}{\pi}\int_0^{1/2\pi}\frac{d\theta}{\sqrt{1-\alpha^2(\cos^2\theta+\chi\sin^2\theta)^2}}, \quad \alpha = \left(\frac{Ehk_2^2}{\rho\omega^2}\right)^{1/2}, \quad \chi = \frac{k_1}{k_2} \tag{2.17}$$

Integration in this case extends over that interval of values of θ where the integrand is real. The function $H(\alpha,\chi)$ was introduced in [2] and has a clear mechanical meaning. It is equal to the ratio of the natural frequencies of the shell for a given ω to its value as $\omega \to \infty$. By a standard device one can express H in terms of elliptic integrals in normal form. For $\chi < 1$ the result of this operation is as follows:

$$H(\alpha,\chi) = \begin{cases} 0 & (\alpha^{-1} < \chi) \\ \sqrt{2}\pi^{-1}[\alpha(1-\chi)]^{-1/2}K\left(\sqrt{1/2\alpha^{-1}(1-\chi)^{-1}(1+\alpha)(1-\alpha\chi)}\right) & (\chi < \alpha^{-1} < 1) \\ 2\pi^{-1}[(1+\alpha)(1-\alpha\chi)]^{-1/2}K(2\alpha(1+\alpha)^{-1}(1-\alpha\chi)^{-1}(1-\chi)) & (\alpha^{-1} > 1) \end{cases} \tag{2.18}$$

where $K(\ )$ is a complete elliptic integral of the first kind. For a plate, $H = 1$; for a spherical shell

$$H(\alpha,1) = \begin{cases} 0 & (\alpha^{-1} < 1) \\ (1-\alpha^2)^{-1/2} & (\alpha^{-1} > 1) \end{cases} \tag{2.19}$$

The results of computing $H(\alpha,\chi)$ for various values of α and χ are shown in Fig.1.

3. Computations similar to the above allow one to obtain the spectral densities of stresses in the shell. As shown in [5], stresses associated with flexure of the shell are given by Formulas

$$\sigma_x = -\frac{6D}{h^2}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)$$
$$\sigma_y = -\frac{6D}{h^2}\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \tag{3.1}$$

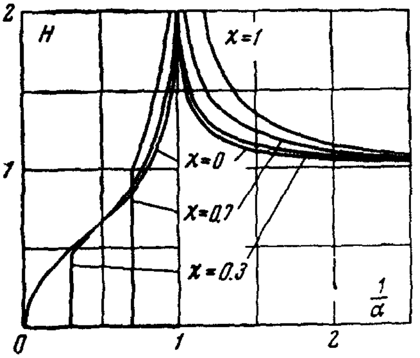

Fig. 1

We limit ourselves here to the consideration of the single stress σ_x. Introducing into (3.1) the spectral representation of the deflection as given by Formulas (1.3) and (1.4), we find the spectral representation of the stress,

$$\sigma_x = \frac{6D}{h^2}\iiint_{-\infty}^{\infty}\frac{(\lambda^2+\nu\mu^2)\,e^{i(\omega t+\lambda x+\mu y)}V(\omega,\lambda,\mu)\,d\omega d\lambda d\mu}{\Omega(\omega,\lambda,\mu)} \tag{3.2}$$

Let us confine ourselves to examining the stress at a single point of the shell. As above, we find the spectral density of this stress,

$$\Phi_{\sigma_x}(\omega) = \left(\frac{6D}{h^2}\right)^2\iint_{-\infty}^{\infty}\frac{(\lambda^2+\nu\mu^2)^2\,S_p(\omega,\lambda,\mu)\,d\lambda d\mu}{|\Omega(\omega,\lambda,\mu)|^2} \tag{3.3}$$

With a load of the wave type, the quantity S_p is of the form (2.3) and computation of the integral in Formula (3.3) yields the following result:

$$\Phi_{\sigma_x}(\omega) = \left(\frac{6D}{h^2}\right)^2 \left(\frac{\omega}{c}\right)^4 \frac{(\cos^2\gamma + \nu\sin^2\gamma)^2\,\Phi_p(\omega)}{|D_k[(\omega/c)^4 = (Eh/D)(k_2\cos^2\gamma + k_1\sin^2\gamma)^2] - \rho\omega^2|^2} \qquad (3.4)$$

where $\Phi_p(\omega)$ is the spectral density of the pressure at a single point on the shell surface. A similar computation gives us the spectral density of the stress σ_y and the correlative spectral density of σ_x and σ_y

$$\Phi_{\sigma_y}(\omega) = \left(\frac{\sin^2\gamma + \nu\cos^2\gamma}{\cos^2\gamma + \nu\sin^2\gamma}\right)^2 \Phi_{\sigma_x}, \qquad \Phi_{\sigma_x\sigma_y}(\omega) = \frac{\sin^2\gamma + \nu\cos^2\gamma}{\cos^2\gamma + \nu\sin^2\gamma}\Phi_{\sigma_x} \qquad (3.5)$$

Now let us find the spectral densities of the stresses for the case where the load is a three-dimensional white noise. We introduce S_p from Formula (2.5) into (3.3) and convert to the interogation variables (2.7),

$$\Phi_{\sigma_x}(\omega) = \left(\frac{6D}{h^2}\right)^2 \frac{\Psi(\omega)}{2} \int_0^{2\pi} (\cos^2\theta + \nu\sin^2\theta)^2\,d\theta \int_0^{\infty} \frac{z^2 dz}{|D_k[z^2 + D^{-1}d^2(\theta)] - \rho\omega^2|} \qquad (3.6)$$

where $d^2(\theta)$ is as in (2.12). The integral over z in the above expression may be computed.

With little friction, i.e. with small ψ, its expression is

$$\frac{\pi[\omega^2 - (Eh/\rho)(k_2\cos^2\theta + k_1\sin^2\theta)^2]^{1/2}}{2D\sqrt{D\rho}\,|\psi|\,\omega^2} \qquad (3.7)$$

If the radical becomes meaningless, the integral is equal to zero. Introducing (3.7) into (3.6), we obtain

$$\Phi_{\sigma_x}(\omega) = \frac{9\pi^2}{4h^4}\frac{\sqrt{D}}{\rho}\frac{\Psi(\omega)}{|\psi\omega|}M_1(\nu, \alpha, \chi) \qquad (3.8)$$

$$M_1(\nu, \alpha, \chi) = \frac{16}{\pi}\int_0^{\frac{1}{2}\pi} (\cos^2\theta + \nu\sin^2\theta)^2 \sqrt{1 - \alpha^2(\cos^2\theta + \chi\sin^2\theta)^2}\,d\theta \qquad (3.9)$$

The values of α and χ are given by Formulas (2.19). Integration in Formula (3.9) is over that interval of values of θ where the integrand has meaning. In exactly the same way we obtain the spectral density of the stress σ_y and the correlative spectral density of σ_x and σ_y,

$$\Phi_{\sigma_y} = \Phi_{\sigma_x}M_2/M_1, \qquad \Phi_{\sigma_x\sigma_y} = \Phi_{\sigma_x}M_3/M_1 \qquad (3.10)$$

where

$$M_2 = \frac{16}{\pi}\int_0^{\frac{1}{2}\pi} (\sin^2\theta + \nu\cos^2\theta)^2 \sqrt{1 - \alpha^2(\cos^2\theta + \chi\sin^2\theta)^2}\,d\theta$$

$$M_3 = \frac{16}{\pi}\int_0^{\frac{1}{2}\pi} (\cos^2\theta + \nu\sin^2\theta)(\sin^2\theta + \nu\cos^2\theta) \sqrt{1 - \alpha^2(\cos^2\theta + \chi\sin^2\theta)^2}\,d\theta \qquad (3.11)$$

As before, integration is carried out only over that interval of values of θ where the integrand is real.

The results of the computations are as follows:

for a plate ($\alpha \to 0$),

$$M_1 = M_2 = 3 + 2\nu + 3\nu^2, \quad M_3 = 1 + 6\nu + \nu^2 \qquad (3.12)$$

for a spherical shell,

$$M = M_2 = (3 + 2\nu + 3\nu^2)\sqrt{1 - \alpha^2} \qquad M_3 = (1 + 6\nu + \nu^2)\sqrt{1 - \alpha^2} \qquad (3.13)$$

For $\alpha > 1$ the values of M are equal to zero.

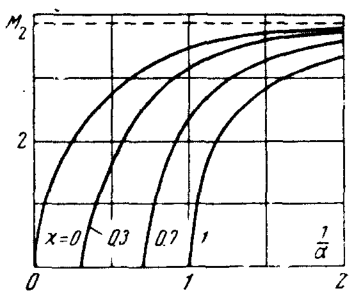

Fig. 2

In general, for $1/\alpha < \chi$ the values of all the M are equal to zero, since in this case the radical in (3.9), (3.11) becomes meaningless. For $1/\alpha > \chi$ integrals (3.9) and (3.11) can be expressed in terms of complete elliptic integrals in normal form. These expressions are cumbersome and difficult to compute, however. It is simpler to integrate numerically. The results of computing M_2 are given in Fig. 2. The values of M_1 and M_3 for $\chi < 1$ turn out to be less then M_2 for each α. For a spherical shell Formulas (3.8), (3.10) and (3.13) yield

$$\Phi_{\sigma_x}(\omega) = \Phi_{\sigma_y}(\omega) = \frac{9\pi^2}{4h^4}\frac{\sqrt{D}}{\sqrt{\rho}}(3 + 2\nu + 3\nu^2)\frac{\Psi(\omega)}{|\psi\omega|}\left(1 - \frac{Ehk^2}{\rho\omega^2}\right)^{1/2} \tag{3.14}$$

With a negative radicand in (3.14) $\Phi = 0$.

The corresponding formula for a plate is obtained from (3.14) for $k = 0$. In this case it coincides to all intents and perposes with the result which Bolotin [2] obtained by means of the asymptotic method.

4. Let us find the spectral density of stresses near the fixed edge of a shell. To simplify matters, let us limit our discussion to the case of a spherical shell $y > 0$. The vibrations of the shell are described by Equations (1.1) for $k_1 = k_2 = k$,

$$D\left[1 + R\left(\frac{d}{dt}\right)\right]\triangle\triangle w - k\triangle\varphi + \rho\frac{\partial^2 w}{\partial t^2} = p(t, x, y)$$
$$Eh\left[1 + R\left(\frac{d}{dt}\right)\right]k\triangle w + \triangle\triangle\varphi = 0 \tag{4.1}$$

Let us assume that the fixed edge is $y = 0$, where the conditions

$$u = v = w = \partial w / \partial y \tag{4.2}$$

must be fulfilled.

As stated in the introduction, disturbances in the stress condition of the shell must decrease with distance from the shell edge ($y \to \infty$) . We limit ourselves to finding the bending stress in the fixed edge of the shell, which may be written as

$$\sigma_y = -6Dh^{-2}\triangle w \tag{4.3}$$

The equation for determining w is obtained by eliminating φ from system (4.1),

$$D\left[1 + R\left(\frac{d}{dt}\right)\right]\left(\triangle\triangle w + \frac{Ehk^2}{D}w\right) + \rho\frac{\partial^2 w}{\partial t^2} = p(t, x, y) \tag{4.4}$$

The tangential displacements u and v will not be determined, since they do not appear in (4.3), and the normal deflection w can be determined independently of u and v by virtue of (4.2) and (4.4).

We will attempt to solve (4.4) with p as defined by Formula (1.2) in the form

$$w = \iiint_{-\infty}^{\infty}\frac{e^{i(\omega t + \lambda x)}V(\omega, \lambda, \mu)}{D_k[(\lambda^2 + \mu^2)^2 + Ehk^2/D] - \rho\omega^2}[e^{i\mu y} + F(y)]\,d\omega d\lambda d\mu \tag{4.5}$$

where $F(y)$ satisfies Equation

$$\left\{D_k\left[\left(-\lambda^2 + \frac{d^2}{dy^2}\right)^2 + \frac{Ehk^2}{D}\right] - \rho\omega^2\right\}F(y) = 0 \tag{4.6}$$

Its solution, which vanishes with increasing y , is of the form

$$F(y) = Ae^{-\beta_1 y} + Be^{-\beta_2 y} \tag{4.7}$$

where

$$\beta_{1,2} = \left[\lambda^2 \pm \left(\frac{\rho\omega^2}{D_k} - \frac{Ehk^2}{D}\right)^{1/2}\right]^{1/2} \tag{4.8}$$

In this expression we take that branch of the outer radical on which the real portion of the radical is positive. Due to energy dissipation, such β_1 and β_2 always exist.

Satisfaction of the boundary conditions (4.2) at the fixed edge of the shell yields the following values for the constants A and B :

$$A = \frac{\beta_2 + i\mu}{\beta_1 - \beta_2}, \qquad B = \frac{\beta_1 + i\mu}{\beta_2 - \beta_1} \tag{4.9}$$

These can be used to find the value of the bending stress in the fixed edge of the shell ($y = 0$),

$$\sigma_y = \frac{6D}{h^2} \iiint_{-\infty}^{\infty} \frac{e^{i(\omega t + \lambda x)} V(\omega, \lambda, \mu)}{D_k [(\lambda^2 + \mu^2)^2 + Fhk^2 / D] - \rho\omega^2} [\mu^2 - \beta_1\beta_2 - i\mu(\beta_1 + \beta_2)]\, d\omega d\lambda d\mu \tag{4.10}$$

The correlation function of the bending stress dependent solely on the time interval τ is of the form

$$K_{\sigma_y}(\tau) = \left(\frac{6D}{h^2}\right)^2 \iiint_{-\infty}^{\infty} \frac{R(\omega, \lambda, \mu)\, e^{i\omega\tau} S_p(\omega, \lambda, \mu)\, d\omega d\lambda d\mu}{|\, D_k [(\lambda^2 + \mu^2)^2 + Ehk^2 / D] - \rho\omega^2 \,|^2} \tag{4.11}$$

$$R(\omega, \lambda, \mu) = |\, \mu^2 - \beta_1\beta_2 - i\mu(\beta_1 + \beta_2) \,|^2 \tag{4.12}$$

The values of R for various relationships between its arguments and parameters are as follows:

$$R = \begin{cases} (\lambda^2 + \mu^2)^2 & \text{for } n < 0 \\ (\lambda^2 + \mu^2)^2 - m & \text{for } 0 < m < \lambda^4 \qquad \left(m = \dfrac{\rho\omega^2 - Ehk^2}{D}\right) \\ \left(\mu \operatorname{sign}\omega + \sqrt{\sqrt{m} - \lambda^2}\right)^2 (\lambda^2 + \mu^2 + \sqrt{m}) & \text{for } m > \lambda^4 \end{cases} \tag{4.13}$$

In these computations we have neglected terms of the order of magnitude of ψ. With the aid of Expression (4.11) we find the spectral density of the stress in the fixed edge,

$$\Phi_{\sigma_y}(\omega) = \left(\frac{6D}{h^2}\right)^2 \iint_{-\infty}^{\infty} \frac{R(\omega, \lambda, \mu)\, S_p(\omega, \lambda, \mu)\, d\lambda d\mu}{|\, D_k [(\lambda^2 + \mu^2)^2 + Ehk^2 / D] - \rho\omega^2 \,|^2} \tag{4.14}$$

We next compute the spectral densities of stresses in the fixed edge for two types of external loads.

With an external load of the wave type the spectral density of the load is of the form (2.3) and computation of the integral in (4.14) yields

$$\Phi_{\sigma_y}(\omega) = \left(\frac{6D}{h^2}\right)^2 \left(\frac{\omega}{c}\right)^4 \frac{\Phi_p(\omega)\, G}{|\, D_k [(\omega / c)^4 + Ehk^2 / D] - \rho\omega^2 \,|^2} \tag{4.15}$$

where G is given by

$$G = \left(\frac{c}{\omega}\right)^4 R\left(\omega, \ \frac{\omega}{c}\cos\gamma, \quad \frac{\omega}{c}\sin\gamma\right) \tag{4.16}$$

In accordance with (4.13) and (4.14), we have the following expressions for G in accordance with the values of the frequency ω :

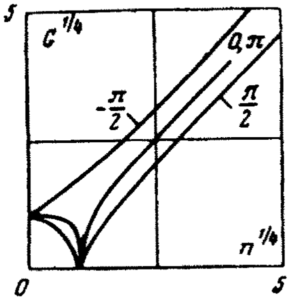

Fig. 3

$$G(n) = \begin{cases} 1 & \text{for } n < 0 \\ 1 - n & \text{for } 0 < n < \cos^4\gamma \\ \left(-\sin\gamma + \sqrt{\sqrt{n} - \cos^2\gamma}\right)^2 (1 + \sqrt{n}) & \text{for } n > \cos^4\gamma \end{cases} \tag{4.17}$$

$$\left(n = \left(\frac{c}{\omega}\right)^4 \frac{\rho\omega^2 - Ehk^2}{D}\right)$$

G represents the ratio of the spectral density of stresses in the fixed edge of a spherical shell to the spectral density of the maximum stress $\Phi_\sigma(\omega)$ in the interior of the shell,

$$\Phi_{\sigma_y}(\omega) = \Phi_\sigma(\omega)\, G, \qquad \Phi_\sigma(\omega) = \left(\frac{6D}{h^2}\right)^2 \left(\frac{\omega}{c}\right)^4 \frac{\Phi_p(\omega)}{|D_k[(\omega/c)^4 + Ehk^2/D] - \rho\omega^2|^2} \tag{4.18}$$

Fig.3 shows the dependence of $G^{1/4}$ on $n^{1/4}$ for several different values of $\gamma = -\frac{1}{2}\pi,\ 0,\ \pi,\ \frac{1}{2}\pi$. If the load applied to the shell is three-dimensional white noise, the spectral density is of the form (2.5). Substituting it into (4.14) and converting to the new integration variables (2.7), we obtain

$$\Phi_{\sigma_y}(\omega) = \left(\frac{6D}{h^2}\right)^2 \frac{\Psi(\omega)}{2} \int_0^{2\pi} d\theta \int_0^\infty \frac{R(\omega,\ \sqrt{z}\cos\theta,\ \sqrt{z}\sin\theta)\, dz}{|D_k(z^2 + Ehk^2/D) - \rho\omega^2|^2} \tag{4.19}$$

The integral over z can be computed approximately by making use of the fact that ψ is small. In this case the principal contribution to the value of the integral is from the values of the integrand in the region close to that value of z where the denominator of the integrand attains a minimum, i.e. for z close to

$$z^* = \sqrt{(\rho\omega^2 - Ehk^2)/D} \tag{4.20}$$

Hence, in computing the integral over z in (4.20) R can be replaced by its value for $z = z^*$. One then obtains the following expression for the integral over z :

$$A = R(\omega,\ \sqrt{z^*}\cos\theta,\ \sqrt{z^*}\sin\theta) \int_0^\infty \frac{dz}{|D_k(z^2 + Ehk^2/D) - \rho\omega^2|^2} \tag{4.21}$$

But this integral has already been computed (see (2.17)). We also note that under conditions (2.7), (4.20) and for $\omega^2 > Ehk^2/\rho$ we have

$$(\rho\omega^2 - Ehk^2)/D > \lambda^4 \tag{4.22}$$

Hence, in (4.21) we must take the bottom expression for R in Formula (4.13).

Introducing λ and μ from Formulas (2.7) and (4.20), we obtain

$$R(\omega,\ \sqrt{z^*}\cos\theta,\ \sqrt{z^*}\sin\theta) = \begin{cases} 8(\rho\omega^2 - Ehk^2)D^{-1}\sin^2\theta & \text{for } \omega\sin\theta > 0 \\ 0 & \text{for } \omega\sin\theta < 0 \end{cases} \tag{4.23}$$

Substituting (4.23) and (2.17) into (4.21) and then introducing the expression for the integral over z into (4.19), we arrive at the following final formula:

$$\Phi_{\sigma_y} = \frac{36}{h^4} \frac{\pi^2\Psi(\omega)}{|\psi\omega|} \left(\frac{D}{\rho}\right)^{1/2} \left(1 - \frac{Ehk^2}{\rho\omega^2}\right)^{1/2} \tag{4.24}$$

With a negative radicand in (4.24) Φ is equal to zero.

Comparing (4.24) with (3. 4), we find that the ratio of the spectral density of stresses in the fixed edge of the shell to its value far away from the edge is equal to

$$16 / (3 + 2\nu + 3\nu^2) \approx 4.14 \quad \text{for} \quad \nu = 0.3$$

A similar result for the dispersion of stresses in a plate was previously obtained by Bolotin [2].

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